

# Algebraic Renormalization of Parity-Preserving QED<sub>3</sub> Coupled to Scalar Matter I: Unbroken Case

*O. M. Del Cima\**, *D. H. T. Franco<sup>†</sup>*, *J. A. Helayël-Neto*  
*and O. Piguet<sup>‡§</sup>*

Centro Brasileiro de Pesquisas Físicas (CBPF)  
 Departamento de Teoria de Campos e Partículas (DCP)  
 Rua Dr. Xavier Sigaud, 150 - Urca  
 22290-180 - Rio de Janeiro - RJ - Brazil.

UGVA-DPT-1996-09-950

## Abstract

In this letter the algebraic renormalization method, which is independent of any kind of regularization scheme, is presented for the parity-preserving QED<sub>3</sub> coupled to scalar matter in the symmetric regime, where the scalar assumes vanishing vacuum expectation value,  $\langle\varphi\rangle=0$ . The model shows to be stable under radiative corrections and anomaly free.

The study of gauge field theories in 3 space-time dimensions [1] has been well-supported by a possible field-theoretical approach to describe some Condensed Matter phenomena, such as High- $T_c$  Superconductivity and Quantum Hall Effect [2, 3]. Some Abelian models have been proposed in this direction, namely, the QED<sub>3</sub> and  $\tau_3$ QED<sub>3</sub> [4, 5].

One of the interesting properties of 3-dimensional gauge field theories is the Landau gauge finiteness of non-Abelian Chern-Simons theories [6].

The confinement of massive electrons in 3 space-time dimensions is a remarkable characteristic of this lower dimensional space [7]. Recently, it was shown by using the Bethe-Salpeter equations that in a parity-preserving QED<sub>3</sub> there are bound states in electron-positron systems, positronium states [8].

In a recent work [9], a parity-preserving QED<sub>3</sub> with spontaneous breaking of a local  $U(1)$ -symmetry was proposed. The breakdown is accomplished by a sixth-power potential. It was shown that electrons scattered in  $D=1+2$  can experience a mutual attractive interaction, depending on their spin states, where the intermediate bosons involved in

---

\*E-mail: oswaldo@cbpfsu1.cat.cbpf.br .

<sup>†</sup>E-mail: franco@cbpfsu1.cat.cbpf.br .

<sup>‡</sup>On leave of absence from *Département de Physique Théorique, Université de Genéve, 24 quai E. Ansermet, CH-1211 Genève 4, Switzerland.* E-mail: piguet@sc2a.unige.ch .

<sup>§</sup>Supported in part by the *Swiss National Science Foundation*.

such processes are a massive vector meson and a Higgs scalar. This attractive scattering potential comes from processes in which the electrons are correlated in momentum space with opposite spin polarizations (*s*-wave state).

One has still to study the renormalizability of this model, with the  $U(1)$  gauge invariance spontaneously broken as explained above. However, the present letter is dedicated to the preliminary task of doing that for the simpler case of unbroken gauge invariance. In this symmetric phase, the gauge boson remains massless. The same should occur for the fermion, since, in the broken phase, its mass is completely generated by the Higgs mechanism.

But since a massless spinor might cause infrared singularities due to the presence of super-renormalizable vertices involving the massless fields, we will add a fermion mass term in order to avoid this problem – which anyhow will not appear in the physically interesting broken phase, where the fermion is anyhow massive.

After a very brief summary of the model, we will show that its parametrization is stable under small perturbations. This, together with the proof of the absence of anomalies given in the final part of the paper, will mean the multiplicative renormalizability of the theory.

The study of the renormalizability of the broken phase will be presented in a forthcoming paper [10].

The gauge invariant action for the parity-preserving  $\text{QED}_3$ <sup>1</sup> coupled to scalar matter [9] in the  $U(1)$ -symmetric regime,  $\langle\varphi\rangle=0$ , is given by:

$$\Sigma_{inv} = \int d^3x \left\{ -\frac{1}{4}F^{mn}F_{mn} + i\bar{\psi}_+ \not{D}\psi_+ + i\bar{\psi}_- \not{D}\psi_- - m_0(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-) + \right. \\ \left. - y(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-)\varphi^*\varphi + D^m\varphi^*D_m\varphi - \mu^2\varphi^*\varphi - \frac{\zeta}{2}(\varphi^*\varphi)^2 - \frac{\lambda}{3}(\varphi^*\varphi)^3 \right\} , \quad (1)$$

where the mass dimensions of the parameters  $m_0$ ,  $\mu$ ,  $\zeta$ ,  $\lambda$  and  $y$  are respectively 1, 1, 1, 0 and 0. The form of the potential is chosen such as to ensure the symmetric regime, where  $\langle\varphi\rangle=0$ . Imposing that it must be bounded from below and yield only stable vacua, we get the following conditions on the parameters:

$$\lambda > 0 , \quad \zeta < 0 \quad \text{and} \quad \mu^2 > \frac{3}{16}\frac{\zeta^2}{\lambda} . \quad (2)$$

The covariant derivatives are defined as follows:

$$\not{D}\psi_{\pm} \equiv (\not{\partial} + iqg\not{A})\psi_{\pm} \quad \text{and} \quad D_m\varphi \equiv (\partial_m + iQgA_m)\varphi , \quad (3)$$

where  $g$  is a coupling constant with dimension of  $(\text{mass})^{\frac{1}{2}}$ , and  $q$  and  $Q$  are the  $U(1)$ -charges of the fermions and scalar, respectively. In the action (1),  $F_{mn}$  is the usual field strength for  $A_m$ ,  $\psi_+$  and  $\psi_-$  are two kinds of fermions (the  $\pm$  subscripts refer to their spin sign [11]) and  $\varphi$  is a complex scalar. It should be noticed that in the action (1) a parity-preserving mass term for  $\psi_+$  and  $\psi_-$  has been added to the original action of ref. [9] in order to avoid potential IR divergences which may be caused by the super-renormalizable interactions.

---

<sup>1</sup>The metric adopted throughout this work is  $\eta_{mn} = (+, -, -)$ ;  $m, n = (0, 1, 2)$ . Note that slashed objects mean contraction with  $\gamma$ -matrices. The latter are taken as  $\gamma^m = (\sigma_x, i\sigma_y, -i\sigma_z)$ .

The complete action,  $\Sigma$ , we are considering here, is given by:

$$\Sigma = \Sigma_{inv} + \Sigma_{gf} + \Sigma_{ext} , \quad (4)$$

where  $\Sigma_{gf}$  is the gauge-fixing action and  $\Sigma_{ext}$  is the action for the external sources:

$$\Sigma_{gf} = \int d^3x \left\{ B \partial^m A_m + \frac{\xi}{2} B^2 + \bar{c} \square c \right\} , \quad (5)$$

$$\Sigma_{ext} = \int d^3x \left\{ \bar{\Omega}_+ s\psi_+ - \bar{\Omega}_- s\psi_- - s\bar{\psi}_+ \Omega_+ + s\bar{\psi}_- \Omega_- + \rho^* s\varphi + s\varphi^* \rho \right\} . \quad (6)$$

The gauge condition, the ghost equation and the antighost equation [12] for (4) read

$$\frac{\delta \Sigma}{\delta B} = \partial^m A_m + \xi B , \quad (7.a)$$

$$\frac{\delta \Sigma}{\delta \bar{c}} = \square c , \quad (7.b)$$

$$-i \frac{\delta \Sigma}{\delta c} = \Delta_{\text{class}} , \quad \text{with:} \quad (7.c)$$

$$\Delta_{\text{class}} = i \square \bar{c} + q \bar{\Omega}_+ \psi_+ - q \bar{\Omega}_- \psi_- + q \bar{\psi}_+ \Omega_+ - q \bar{\psi}_- \Omega_- - Q \rho^* \varphi - Q \varphi^* \rho .$$

Note that the right-hand sides being linear in the quantum fields, will not be submitted to renormalization. The  $\text{QED}_3$ -action<sup>2</sup> (4) is invariant under the reflexion symmetry  $P$ , whose action on the fields and external sources is fixed as below:

$$\begin{aligned} x_m &\xrightarrow{P} x_m^P = (x_0, -x_1, x_2) , \\ \psi_{\pm} &\xrightarrow{P} \psi_{\pm}^P = -i\gamma^1 \psi_{\mp} , \quad \bar{\psi}_{\pm} \xrightarrow{P} \bar{\psi}_{\pm}^P = i\bar{\psi}_{\mp} \gamma^1 , \\ A_m &\xrightarrow{P} A_m^P = (A_0, -A_1, A_2) , \\ \phi &\xrightarrow{P} \phi^P = \phi , \quad \phi = \varphi, c, \bar{c}, B , \\ \Omega_{\pm} &\xrightarrow{P} \Omega_{\pm}^P = -i\gamma^1 \Omega_{\mp} , \quad \bar{\Omega}_{\pm} \xrightarrow{P} \bar{\Omega}_{\pm}^P = i\bar{\Omega}_{\mp} \gamma^1 , \\ \rho &\xrightarrow{P} \rho^P = \rho . \end{aligned} \quad (8)$$

The ultraviolet and infrared dimensions<sup>3</sup>,  $d$  and  $r$  respectively, as well as the ghost numbers,  $\Phi\Pi$ , and the Grassmann parity,  $GP$ , of all fields and sources are collected in Table 1.

The BRS transformations are defined by:

$$\begin{aligned} s\varphi &= iQc\varphi , \quad s\varphi^* = -iQc\varphi^* , \\ s\psi_{\pm} &= iq\bar{c}\psi_{\pm} , \quad s\bar{\psi}_{\pm} = -iq\bar{c}\bar{\psi}_{\pm} , \\ sA_m &= -\frac{1}{g} \partial_m c , \quad sc = 0 , \\ s\bar{c} &= \frac{1}{g} B , \quad sB = 0 , \end{aligned} \quad (9)$$

<sup>2</sup> For more details about  $\text{QED}_3$  and  $\tau_3\text{QED}_3$  as well as their applications, and some peculiarities of parity and time-reversal in  $D=1+2$ , see refs. [1, 4, 5].

<sup>3</sup> We have to use a subtraction scheme which takes care of the presence of both massive and massless fields, subtracting off the UV divergences without introducing spurious IR singularities. Such a scheme is the one of Lowenstein and Zimmermann [13, 14]. The UV and IR dimensions mentioned here are those which are involved in this formalism. The terms in the action, as well as all counterterms, are constrained to have UV dimension  $\leq 3$  and IR dimension  $\geq 3$ .

	$A_m$	$\varphi$	$\psi_{\pm}$	$c$	$\bar{c}$	$B$	$\rho$	$\Omega_{\pm}$
$d$	1/2	1/2	1	0	1	3/2	5/2	2
$r$	1/2	3/2	3/2	0	1	3/2	5/2	2
$\Phi\Pi$	0	0	0	1	-1	0	-1	-1
$GP$	0	0	1	1	1	0	1	0

Table 1: UV and IR dimensions,  $d$  and  $r$ , ghost numbers,  $\Phi\Pi$ , and Grassmann parity,  $GP$ .

where  $c$  is the ghost,  $\bar{c}$  is the antighost and  $B$  is the Lagrange multiplier field.

The BRS invariance of the action is expressed in a functional way by the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = 0 \quad , \quad (10)$$

where the Slavnov-Taylor operator  $\mathcal{S}$  is defined, acting on an arbitrary functional  $\mathcal{F}$ , by

$$\begin{aligned} \mathcal{S}(\mathcal{F}) = \int d^3x \left\{ -\frac{1}{g} \partial^m c \frac{\delta \mathcal{F}}{\delta A^m} + \frac{1}{g} B \frac{\delta \mathcal{F}}{\delta \bar{c}} + \frac{\delta \mathcal{F}}{\delta \bar{\Omega}_+} \frac{\delta \mathcal{F}}{\delta \psi_+} - \frac{\delta \mathcal{F}}{\delta \bar{\Omega}_-} \frac{\delta \mathcal{F}}{\delta \psi_-} - \frac{\delta \mathcal{F}}{\delta \Omega_+} \frac{\delta \mathcal{F}}{\delta \bar{\psi}_+} + \frac{\delta \mathcal{F}}{\delta \Omega_-} \frac{\delta \mathcal{F}}{\delta \bar{\psi}_-} + \right. \\ \left. + \frac{\delta \mathcal{F}}{\delta \rho^*} \frac{\delta \mathcal{F}}{\delta \varphi} - \frac{\delta \mathcal{F}}{\delta \rho} \frac{\delta \mathcal{F}}{\delta \varphi^*} \right\} \quad . \end{aligned} \quad (11)$$

The corresponding linearized Slavnov-Taylor operator reads

$$\begin{aligned} \mathcal{S}_{\mathcal{F}} = \int d^3x \left\{ -\frac{1}{g} \partial^m c \frac{\delta}{\delta A^m} + \frac{1}{g} B \frac{\delta}{\delta \bar{c}} + \frac{\delta \mathcal{F}}{\delta \bar{\Omega}_+} \frac{\delta}{\delta \psi_+} - \frac{\delta \mathcal{F}}{\delta \bar{\Omega}_-} \frac{\delta}{\delta \psi_-} + \frac{\delta \mathcal{F}}{\delta \psi_+} \frac{\delta}{\delta \bar{\Omega}_+} - \frac{\delta \mathcal{F}}{\delta \psi_-} \frac{\delta}{\delta \bar{\Omega}_-} + \right. \\ \left. - \frac{\delta \mathcal{F}}{\delta \Omega_+} \frac{\delta}{\delta \bar{\psi}_+} + \frac{\delta \mathcal{F}}{\delta \Omega_-} \frac{\delta}{\delta \bar{\psi}_-} - \frac{\delta \mathcal{F}}{\delta \bar{\psi}_+} \frac{\delta}{\delta \Omega_+} + \frac{\delta \mathcal{F}}{\delta \bar{\psi}_-} \frac{\delta}{\delta \Omega_-} + \frac{\delta \mathcal{F}}{\delta \rho^*} \frac{\delta}{\delta \varphi} + \frac{\delta \mathcal{F}}{\delta \varphi} \frac{\delta}{\delta \rho^*} + \right. \\ \left. - \frac{\delta \mathcal{F}}{\delta \rho} \frac{\delta}{\delta \varphi^*} - \frac{\delta \mathcal{F}}{\delta \varphi^*} \frac{\delta}{\delta \rho} \right\} \quad . \end{aligned} \quad (12)$$

The following nilpotency identities hold:

$$\mathcal{S}_{\mathcal{F}} \mathcal{S}(\mathcal{F}) = 0 \quad , \quad \forall \mathcal{F} \quad , \quad (13.a)$$

$$\mathcal{S}_{\mathcal{F}} \mathcal{S}_{\mathcal{F}} = 0 \quad \text{if} \quad \mathcal{S}(\mathcal{F}) = 0 \quad . \quad (13.b)$$

In particular:

$$(\mathcal{S}_{\Sigma})^2 = 0 \quad , \quad (14)$$

since the action  $\Sigma$  obeys the Slavnov-Taylor identity. The operation of  $\mathcal{S}_{\Sigma}$  over the fields and the external sources is given by

$$\begin{aligned} \mathcal{S}_{\Sigma} \phi &= s\phi \quad , \quad \phi = \psi_{\pm}, \bar{\psi}_{\pm}, \varphi, \varphi^*, A_m, c, \bar{c} \text{ and } B \quad , \\ \mathcal{S}_{\Sigma} \bar{\Omega}_+ &= \frac{\delta \Sigma}{\delta \psi_+} \quad , \quad \mathcal{S}_{\Sigma} \bar{\Omega}_- = -\frac{\delta \Sigma}{\delta \psi_-} \quad , \\ \mathcal{S}_{\Sigma} \Omega_+ &= -\frac{\delta \Sigma}{\delta \bar{\psi}_+} \quad , \quad \mathcal{S}_{\Sigma} \Omega_- = \frac{\delta \Sigma}{\delta \bar{\psi}_-} \quad , \\ \mathcal{S}_{\Sigma} \rho^* &= \frac{\delta \Sigma}{\delta \varphi} \quad , \quad \mathcal{S}_{\Sigma} \rho = -\frac{\delta \Sigma}{\delta \varphi^*} \quad . \end{aligned} \quad (15)$$

In order to study the stability [15] of the action (4) under the radiative corrections, one has to find the most general counterterm,  $\Sigma^c$ , satisfying the following condition of BRS invariance:

$$\mathcal{S}_\Sigma \Sigma^c = 0 \quad . \quad (16)$$

$\Sigma^c$  is an integrated local polynomial in the fields and its derivatives with UV dimension  $\leq 3$ , IR dimension  $\geq 3$  and with vanishing ghost number. It has to be invariant under the  $P$ -symmetry given by Eqs.(8), and it has also to satisfy the conditions

$$\frac{\delta \Sigma^c}{\delta B} = 0 \quad , \quad \frac{\delta \Sigma^c}{\delta \bar{c}} = 0 \quad , \quad \frac{\delta \Sigma^c}{\delta c} = 0 \quad , \quad (17)$$

which follow from the conditions (7.a – 7.c), and, moreover:

$$W_{\text{rigid}} \Sigma^c = 0 \quad , \quad (18)$$

where  $W_{\text{rigid}}$  is the Ward operator of rigid symmetry defined by

$$W_{\text{rigid}} = \int d^3x \left\{ q\psi_+ \frac{\delta}{\delta \psi_+} + q\psi_- \frac{\delta}{\delta \psi_-} - q\bar{\psi}_+ \frac{\delta}{\delta \bar{\psi}_+} - q\bar{\psi}_- \frac{\delta}{\delta \bar{\psi}_-} + Q\varphi \frac{\delta}{\delta \varphi} - Q\varphi^* \frac{\delta}{\delta \varphi^*} + q\Omega_+ \frac{\delta}{\delta \Omega_+} + q\Omega_- \frac{\delta}{\delta \Omega_-} - q\bar{\Omega}_+ \frac{\delta}{\delta \bar{\Omega}_+} - q\bar{\Omega}_- \frac{\delta}{\delta \bar{\Omega}_-} + Q\rho \frac{\delta}{\delta \rho} - Q\rho^* \frac{\delta}{\delta \rho^*} \right\} \quad . \quad (19)$$

Eq.(18) follows from the rigid  $U(1)$  invariance of the action<sup>4</sup>:

$$W_{\text{rigid}} \Sigma = 0 \quad . \quad (20)$$

We find that the most general invariant counterterm  $\Sigma^c$ , i.e. the most general field polynomial of UV and IR dimensions bounded by  $d \leq 3$  and  $r \geq 3$ , with ghost number zero, respecting  $P$ -symmetry, (8) and the conditions displayed in Eqs.(16), (17) and (18), is given by an arbitrary superposition of the following expressions:

$$\left\{ F^{mn} F_{mn} , \quad i(\bar{\psi}_+ \not{D} \psi_+ + \bar{\psi}_- \not{D} \psi_-) , \quad (\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_-) , \quad (\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_-) \varphi^* \varphi , \quad D^m \varphi^* D_m \varphi , \quad \varphi^* \varphi , \quad (\varphi^* \varphi)^2 , \quad (\varphi^* \varphi)^3 \right\} \quad . \quad (21)$$

The BRS consistency condition in the sector of ghost number zero, given by Eq.(16), constitutes a cohomology problem due to the nilpotency (14) of the linearized Slavnov-Taylor operator (12). Its solution can always be written as a sum of a trivial cocycle  $\mathcal{S}_\Sigma \hat{\Sigma}$ , where  $\hat{\Sigma}$  has ghost number  $-1$ , and a nontrivial part  $\Sigma_{\text{phys}}$  belonging to the cohomology of  $\mathcal{S}_\Sigma$  (12) in the sector of ghost number zero, i.e. which cannot be written as a  $\mathcal{S}_\Sigma$ -variation:

$$\Sigma^c = \Sigma_{\text{phys}} + \mathcal{S}_\Sigma \hat{\Sigma} \quad . \quad (22)$$

---

<sup>4</sup> Rigid invariance itself follows from the antighost equation (7.c) and from the validity of the Slavnov-Taylor identity (10).

One checks indeed that the general invariant counterterm, expanded in the basis (21), admits the representation (22), with

$$\begin{aligned}\Sigma_{\text{phys}} = & z_g \left( g \frac{\partial}{\partial g} - N_A + N_B - 2\xi \frac{\partial}{\partial \xi} \right) \Sigma + z_{m_0} m_0 \frac{\partial \Sigma}{\partial m_0} + \\ & + z_y y \frac{\partial \Sigma}{\partial y} + z_{\mu^2} \mu^2 \frac{\partial \Sigma}{\partial \mu^2} + z_{\zeta} \zeta \frac{\partial \Sigma}{\partial \zeta} + z_{\lambda} \lambda \frac{\partial \Sigma}{\partial \lambda} ,\end{aligned}\quad (23.a)$$

$$\begin{aligned}\mathcal{S}_{\Sigma} \hat{\Sigma} = & \mathcal{S}_{\Sigma} \int d^3x \left[ z_{\psi} \left( \bar{\psi}_+ \Omega_+ - \bar{\Omega}_+ \psi_+ - \bar{\psi}_- \Omega_- + \bar{\Omega}_- \psi_- \right) + z_{\varphi} (\rho^* \varphi - \varphi^* \rho) \right] \\ = & z_{\psi} \left( N_{\psi_+} + N_{\bar{\psi}_+} + N_{\psi_-} + N_{\bar{\psi}_-} - N_{\Omega_+} - N_{\bar{\Omega}_+} - N_{\Omega_-} - N_{\bar{\Omega}_-} \right) \Sigma \\ & + z_{\varphi} (N_{\varphi} + N_{\varphi^*} - N_{\rho} - N_{\rho^*}) \Sigma ,\end{aligned}\quad (23.b)$$

where the counting operators are defined by

$$N_{\phi} = \int d^3x \phi \frac{\delta}{\delta \phi} , \quad \phi = \psi_{\pm}, \bar{\psi}_{\pm}, \Omega_{\pm}, \bar{\Omega}_{\pm}, \varphi, \varphi^*, \rho, \rho^*, A_m \text{ and } B . \quad (24)$$

This way of writing the counterterm makes explicit the separation between the physical counterterms, on the one hand, which amount to the renormalization of the physical masses and coupling constants  $m_0, \mu, g, y, \zeta, \lambda$ , and the trivial ones, on the other hand, which correspond to the unphysical renormalization of the amplitudes of the fields  $\psi_{\pm}$  and  $\varphi$  – the other field renormalizations not being independent. The form of the classical action  $\Sigma$  (4), taken as a  $P$ -invariant solution of the functional identities expressing the various symmetries of the theory, is thus stable under small perturbations, the general solution in a neighbourhood of  $\Sigma$  being obtained through an arbitrary variation of the parametrization.

At the quantum level the vertex functional  $\Gamma$ , which coincides with the classical action (4) at order 0 in  $\hbar$ :

$$\Gamma = \Sigma + \mathcal{O}(\hbar) , \quad (25)$$

has to satisfy the constraints

$$\frac{\delta \Gamma}{\delta B} = \partial^m A_m + \xi B , \quad (26.a)$$

$$\frac{\delta \Gamma}{\delta \bar{c}} = \square c , \quad (26.b)$$

$$-i \frac{\delta \Gamma}{\delta c} = \Delta_{\text{class}} , \quad (26.c)$$

$$W_{\text{rigid}} \Gamma = 0 , \quad (26.d)$$

where  $W_{\text{rigid}}$  has already been defined by equation (19) and Eqs.(26.a – 26.c) are the quantum extension of Eqs.(7.a – 7.c).

According to the Quantum Action Principle [16, 17] the Slavnov-Taylor identity (10) gets a quantum breaking

$$\mathcal{S}(\Gamma) = \Delta \cdot \Gamma = \Delta + \mathcal{O}(\hbar \Delta) , \quad (27)$$

where  $\Delta$  is an integrated local functional with ghost number 1 and dimension 3.

The nilpotency identity (13.a) together with

$$\mathcal{S}_\Gamma = \mathcal{S}_\Sigma + \mathcal{O}(\hbar) \quad (28)$$

implies the following consistency condition for the breaking  $\Delta$ :

$$\mathcal{S}_\Sigma \Delta = 0 \quad . \quad (29)$$

Other constraints on  $\Delta$  follow from the constraints (26.a – 26.d) and from the algebra

$$\frac{\delta \mathcal{S}(\mathcal{F})}{\delta B} - \mathcal{S}_\mathcal{F} \left( \frac{\delta \mathcal{F}}{\delta B} - \partial^m A_m - \xi B \right) = \frac{1}{g} \left( \frac{\delta \mathcal{F}}{\delta \bar{c}} - \square c \right) \quad , \quad (30.a)$$

$$\frac{\delta \mathcal{S}(\mathcal{F})}{\delta \bar{c}} + \mathcal{S}_\mathcal{F} \frac{\delta \mathcal{F}}{\delta \bar{c}} = 0 \quad , \quad (30.b)$$

$$-i \int d^3 x \frac{\delta}{\delta c} \mathcal{S}(\mathcal{F}) + \mathcal{S}_\mathcal{F} \int d^3 x \left( -i \frac{\delta}{\delta c} \mathcal{F} - \Delta_{\text{class}} \right) = W_{\text{rigid}} \mathcal{F} \quad , \quad (30.c)$$

$$W_{\text{rigid}} \mathcal{S}(\mathcal{F}) - \mathcal{S}_\mathcal{F} W_{\text{rigid}} \mathcal{F} = 0 \quad , \quad (30.d)$$

( $\mathcal{F}$  arbitrary functional of ghost number zero) .

These constraints on the breaking  $\Delta$  read:

$$\frac{\delta \Delta}{\delta B} = 0 \quad , \quad (31.a)$$

$$\frac{\delta \Delta}{\delta \bar{c}} = 0 \quad , \quad (31.b)$$

$$\int d^3 x \frac{\delta}{\delta c} \Delta = 0 \quad , \quad (31.c)$$

$$W_{\text{rigid}} \Delta = 0 \quad . \quad (31.d)$$

The Wess-Zumino consistency condition (29) constitutes a cohomology problem like in the zero ghost number case (16). Its solution can always be written as a sum of a trivial cocycle  $\mathcal{S}_\Sigma \hat{\Delta}^{(0)}$ , where  $\hat{\Delta}^{(0)}$  has ghost number 0, and of nontrivial elements belonging to the cohomology of  $\mathcal{S}_\Sigma$  (12) in the sector of ghost number one:

$$\Delta^{(1)} = \hat{\Delta}^{(1)} + \mathcal{S}_\Sigma \hat{\Delta}^{(0)} \quad , \quad (32)$$

where  $\Delta^{(1)}$  must be even under  $P$ -symmetry and obey the conditions imposed by Eqs. (31.a – 31.d). The trivial cocycle  $\mathcal{S}_\Sigma \hat{\Delta}^{(0)}$  can be absorbed into the vertex functional  $\Gamma$  as a noninvariant integrated local counterterm  $-\hat{\Delta}^{(0)}$ . On the other hand, a nonzero  $\Delta^{(1)}$  would represent an anomaly.

Considering the condition (31.c), to be satisfied by (32), it can be concluded that

$$\Delta^{(1)} = \int d^3 x K_m^{(0)} \partial^m c \quad . \quad (33)$$

By analyzing the Slavnov-Taylor operator  $\mathcal{S}_\Sigma$  (12) and the Wess-Zumino consistency condition (29), one sees that the breaking  $\Delta^{(1)}$  has UV and IR dimensions bounded by  $d \leq \frac{7}{2}$

and  $r \geq 2$ . Therefore, the dimensions of  $K_m^{(0)}$  must be bounded by  $d \leq \frac{5}{2}$  and  $r \geq 1$ , it has ghost number 0, and due to Eq.(29) and Eqs. (31.a – 31.b), it must respect the conditions

$$\frac{\delta K_m^{(0)}}{\delta B} = 0 \quad \text{and} \quad \frac{\delta K_m^{(0)}}{\delta \bar{c}} = 0 \quad . \quad (34)$$

Now, rewriting  $K_m^{(0)}$  as a linear combination

$$K_m^{(0)} = \sum_{i=1}^7 a_i K_m^{(0)i} \quad , \quad (35)$$

where

$$\begin{aligned} K_m^{(0)1} &= A_m A^n A_n \quad , \quad K_m^{(0)2} = A_m (A^n A_n)^2 \quad , \quad K_m^{(0)3} = A_m (\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_-) \quad , \\ K_m^{(0)4} &= A_m A^n A_n \varphi^* \varphi \quad , \quad K_m^{(0)5} = A_m (\varphi^* \varphi)^2 \quad , \quad K_m^{(0)6} = A_m \varphi^* \varphi \quad , \\ K_m^{(0)7} &= \bar{\psi}_+ \gamma_m \psi_+ + \bar{\psi}_- \gamma_m \psi_- \quad , \end{aligned} \quad (36)$$

and solving all the conditions it has to fulfil, we can easily show, with the help of Eqs.(15), that there exist local functionals  $\hat{\Delta}^{(0)i}$  such that

$$\int d^3x \ K_m^{(0)i} \partial^m c = \mathcal{S}_\Sigma \hat{\Delta}^{(0)i} \quad , \quad i = 1, \dots, 7 \quad . \quad (37)$$

This means  $\hat{\Delta}^{(1)} = 0$  in (32), which implies the implementability of the Slavnov-Taylor identity to every order through the absorption of the noninvariant counterterm  $-\sum_i a_i \hat{\Delta}^{(0)i}$ .

Of course, invariant counterterms may still be arbitrarily added at each order. However the result of the discussion on the stability of the classical theory shows that these counterterms correspond to a renormalization of the parameters of the theory. Their coefficients have to be fixed by suitable normalization conditions.

In conclusion, we have shown the renormalizability and absence of gauge anomaly for the parity-preserving  $\text{QED}_3$  coupled to scalar matter in the symmetric phase.

## Acknowledgements

The authors express their gratitude to Prof. S.P. Sorella for patient and helpful discussions. One of the authors (O.M.D.C.) dedicates this work to his wife, Zilda Cristina, and to his daughter, Vittoria, who was born in 20 March 1996. O.M.D.C. and D.H.T.F. thank to the *High Energy Section* of the *ICTP - Trieste - Italy*, where this work was done, for the kind hospitality and financial support, and to its Head, Prof. S. Randjbar-Daemi. O.P. thanks the CBPF, the Physics Department of the Federal University of Espírito Santo (Brazil) and his colleagues from both institutions for their very kind hospitality. Thanks are also due to the Head of CFC-CBPF, Prof. A.O. Caride, and Dr. R. Paunov, for encouragement. CNPq-Brazil is acknowledged for invaluable financial help.

## References

- [1] W. Siegel, *Nucl. Phys.* **B156** (1979) 135 ; J. Schonfeld, *Nucl. Phys.* **B185** (1981) 157 ; R. Jackiw and S. Templeton, *Phys. Rev.* **D23** (1981) 2291 ; S. Deser and R. Jackiw, *Phys. Lett.* **B139** (1984) 371 ; S. Deser, R. Jackiw and S. Templeton, *Phys. Rev. Lett.* **48** (1982) 975 and *Ann. Phys. (N.Y.)* **140** (1982) 372.

- [2] J.G. Bednorz and A.K. Müller, *Z. Phys.* **B64** (1986) 189.
- [3] K. Von Klitzing, G. Dorda and M. Peper, *Phys. Rev. Lett.* **45** (1980) 494 ; D. Tsui, H. Störmer and A. Gossard, *Phys. Rev. Lett.* **48** (1982) 1559.
- [4] S. Randjbar-Daemi, A. Salam and J. Strathdee, *Nucl. Phys.* **B340** (1990) 403 and *Int. J. Mod. Phys.* **B5** (1991) 845 ; N. Dorey and N.E. Mavromatos, *Phys. Lett.* **B266** (1991) 163 and *Nucl. Phys.* **B386** (1992) 614.
- [5] R.D. Pisarski, *Phys. Rev.* **D29** (1984) 2423 ; T.W. Appelquist, M. Bowick, D. Karabali and L.C.R. Wijewardhana, *Phys. Rev.* **D33** (1986) 3704 ; R. Mackenzie and F. Wilczek, *Int. J. Mod. Phys.* **A3** (1988) 2827 ; G.W. Semenoff and P. Sodano, *Nucl. Phys.* **B328** (1989) 753 ; G.W. Semenoff and L.C.R. Wijewardhana, *Phys. Rev. Lett.* **63** (1989) 2633 ; A. Kovner and B. Rosenstein, *Mod. Phys. Lett.* **A5** (1990) 2661 ; M.I. Dobroliubov, D. Eliezer, I.I. Kogan, G.W. Semenoff and R.J. Szabo, *Mod. Phys. Lett.* **A8** (1993) 2177.
- [6] F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl. Phys.* **B346** (1990) 313 ; C. Lucchesi and O. Piguet, *Nucl. Phys.* **B381** (1992) 281.
- [7] P. Maris, *Phys. Rev.* **D52** (1995) 6087.
- [8] C.J. Burden, *Nucl. Phys.* **B387** (1992) 419 ; T.W. Allen and C.J. Burden, *Phys. Rev.* **D53** (1996) 5842.
- [9] M.A. De Andrade, O.M. Del Cima and J.A. Helayël-Neto, *Electron-pair condensation in parity-preserving QED<sub>3</sub>*, hep-th/9603054. Talk given at *Quantum Systems: New Trends and Methods 96 - QS96*, Minsk, Belarus.
- [10] O.M. Del Cima, D.H.T. Franco, J.A. Helayël-Neto and O. Piguet, *Algebraic renormalization of parity-preserving QED<sub>3</sub> coupled to scalar matter II: broken case*, UGVA-DPT-1996-10-953.
- [11] B. Binegar, *J. Math. Phys.* **23** (1982) 1511.
- [12] A. Blasi, O. Piguet and S.P. Sorella, *Nucl. Phys.* **B356** (1991) 154.
- [13] J.H. Lowenstein, *Comm. Math. Phys.* **47** (1976) 53.
- [14] O. Piguet, *Renormalisation en théorie quantique de champs* and *Renormalisation des théories de jauge*, lectures of the *Troisième cycle de la physique en Suisse Romande* (1982-1983).
- [15] C. Becchi, A. Rouet and R. Stora, *Comm. Math. Phys.* **42** (1975) 127 and *Ann. Phys. (N.Y.)* **98** (1976) 287 ; O. Piguet and A. Rouet, *Phys. Rep.* **76** (1981) 1.
- [16] O. Piguet and S.P. Sorella, *Algebraic Renormalization*, Lecture Notes in Physics, m28, Springer-Verlag, Berlin, Heidelberg, 1995.
- [17] J.H. Lowenstein, *Phys. Rev.* **D4** (1971) 2281 and *Comm. Math. Phys.* **24** (1971) 1 ; Y.M.P. Lam, *Phys. Rev.* **D6** (1972) 2145 and *Phys. Rev.* **D7** (1973) 2943 ; T.E. Clark and J.H. Lowenstein, *Nucl. Phys.* **B113** (1976) 109.